# ON A CLASS OF SUMS WITH UNEXPECTEDLY HIGH CANCELLATION AND PRIMES IN SHORT INTERVALS

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Abstract. We report on the discovery of a general principle leading to the unexpected cancellation of oscillating sums. It turns out that sums in the class we consider are much smaller than would be predicted by certain probabilistic heuristics. After stating the motivation, and our theorem, we prove a "Pentagonal Number Theorem for the Primes", which counts the number of primes (with von Mangoldt weight) in a set of intervals very precisely.

$$\sum_{0\leq 2\ell<\sqrt{x}} \Psi([e^{c\sqrt{x-(2\ell)^2}},\ e^{c\sqrt{x-(2\ell-1)^2}}]) \ = \Psi(e^{c\sqrt{x}})\left(\frac{1}{2}+O\left(\sqrt[8]{c}\right)\right),$$

where $c=\frac{1}{\sqrt{T}}$ , where $x=\frac{9T\log(T)}{16\alpha^2}$ and $\alpha=1-\sqrt{\frac{2}{2+\pi^2}}$. In fact the result is stronger than one would get using a strong form of the Prime Number Theorem and also the Riemann Hypothesis (where one naively estimates the $\Psi$ function on each of the intervals; however, a less naive argument can give an improvement), since the widths of the intervals are smaller than $\sqrt{x}$, making the Riemann Hypothesis estimate "trivial". This was a joint work with Ernie Croot.